Approximate Equations for Impact Dispersion Resulting from Winds and Deviations in Density

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Theme

FOR accurate prediction of the location of a long-range ballistic missile impact, the effect of the atmosphere's meteorological characteristics (especially density and wind profiles) near the impact point must be considered. Aiming instructions that are given at the firing point are frequently based on a standard density profile and zero wind, with corrections then applied to account for the existing winds and for deviations in density from the standard profile. Such corrections are obtained by use of three- or six-degree-of-freedom trajectory simulations. However, use of a closed-form solution to the equations of motion, if sufficiently simple while still accurate, is more economical, is faster, is more useful in situations where computing capability is limited, and provides helpful insight to the user. The full report presents such a solution, including derivations of the working equations. These equations, their applications, and their limitations are presented herein.

Contents

The dispersion δ_w (ft) resulting from a re-entry wind component (north-south or east-west) and the dispersion δ_ρ (ft) resulting from deviations in density may be estimated using the following equations:

$$\delta_w = -\frac{HI_3}{V_E \sin\gamma_E}\sum_{j=1}^{N} F_{w_j} V_{w_j} \tag{1}$$

$$\delta_\rho = \left(\frac{H}{V_E \sin\gamma_E}\right)^2 \frac{gI_5}{\tan\gamma_E}\sum_{j=1}^{N} F_{\rho_j}\left(\frac{\Delta\rho}{\rho}\right)_j \tag{2}$$

where

F_w, F_ρ = dimensionless weighting factors for wind and density
g, H = constants, 32.2 ft/sec² and 23,000 ft, respectively
I_3, I_5 = functions (Table 1) evaluated at $K_{SL} = -P_{SL}/\beta \sin\gamma_E$
j = index for altitude increment, $j = 1, 2, \ldots, N$ (Table 2)
P_{SL} = ambient pressure at sea level, lb/ft²
V_E = re-entry velocity at altitude $h = 400{,}000$ ft, fps
V_w = north-south or east-west component of wind speed, positive for a wind from West and South, ft/sec
β = vehicle ballistic coefficient (weight W)/(axial force coefficient C_A × reference area S), lb/ft²
$\Delta\rho/\rho$ = dimensionless density deviation (ρ is standard ambient density), positive when perturbed density is greater than standard value
γ_E = angle between the velocity vector and the horizontal at $h = 400{,}000$ ft, a negative value, deg

Equation (1) is evaluated for the east-west and north-south components of δ_w; δ_ρ is an uprange-downrange component. Values of δ_w are positive for impacts east and north of the impact of the zero-wind trajectory; values of δ_ρ are positive for impacts downrange of the impact of the zero-density deviation trajectory. The total dispersion (δ_{SL}, at sea level) is the vector sum of the three components. For the particular cases of constant V_w and $\Delta\rho/\rho$, the summation terms may be replaced by V_w and $\Delta\rho/\rho$. The terms I_3 and I_5 are functions of K_{SL}; K_w and K_ρ are functions of altitude, altitude band width, and K_{SL}. The four functions are evaluated theoretically and empirically in the report. The empirical results, summarized in Tables 1 and 2, were obtained applying Eqs. (1) and (2) as correlation equations to dispersion data[1] computed using a three-degree-of-freedom (3DOF) simulation for a variety of configurations and re-entry conditions (β = 550–2000 lb/ft², V_E = 15,000–23,000 fps, $\gamma_E = -20°$ to $-50°$).

Figure 1 shows that the accuracy of the correlation is good

Table 1 *I* functions

K_{SL}	I_3	I_5	K_{SL}	I_3	I_5
0.5	0.25	0.50	3.5	3.20	11.30
1.0	0.57	1.05	4.0	4.13	17.30
1.5	0.92	1.85	4.5	5.25	27.00
2.0	1.33	3.00	5.0	6.60	41.00
2.5	1.84	4.90	5.5	8.17	61.00
3.0	2.44	7.55	6.0	9.90	92.50

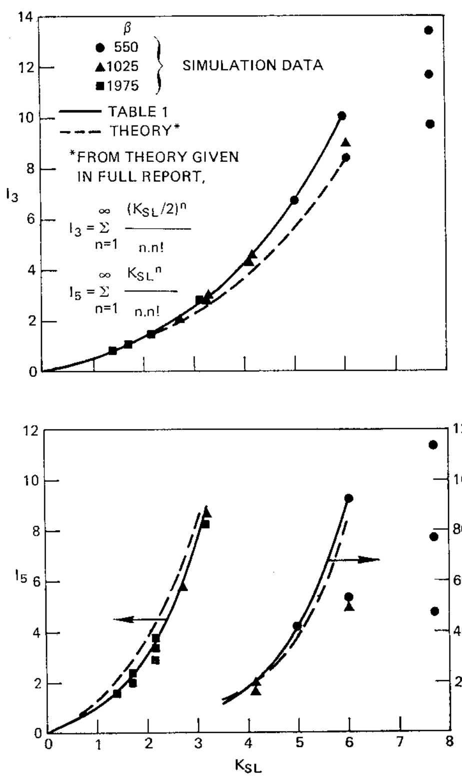

Fig. 1 Comparisons of I_3 and I_5 determined by 3DOF simulations and theory with recommended values (Table 1).

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Table 2 F_w and F_ρ functions for six values of $K_{SL} = -P_{SL}/\beta \sin\gamma_E$

Altitude (thousands of ft)	j	F_w						F_ρ					
		$K_{SL} = 1.5$	2.0	3.0	4.0	5.0	6.0	$K_{SL} = 1.5$	2.0	3.0	4.0	5.0	6.0
160–∞	1	0.007	0.006	0.005	0.004	0.003	0.003	0.004	0.012	0.009	0.006	0.004	0.004
140–160	2	0.005	0.004	0.004	0.002	0.002	0.001	0.014	0.014	0.009	0.006	0.005	0.004
120–140	3	0.010	0.009	0.007	0.006	0.005	0.004	0.027	0.023	0.107	0.012	0.008	0.007
100–120	4	0.025	0.023	0.018	0.013	0.011	0.010	0.053	0.046	0.035	0.025	0.019	0.017
90–100	5	0.021	0.019	0.015	0.011	0.009	0.008	0.045	0.039	0.028	0.021	0.015	0.013
80–90	6	0.030	0.027	0.021	0.017	0.015	0.013	0.055	0.050	0.039	0.029	0.025	0.022
70–80	7	0.045	0.040	0.030	0.024	0.020	0.016	0.069	0.067	0.058	0.045	0.034	0.030
60–70	8	0.064	0.058	0.047	0.035	0.029	0.025	0.088	0.085	0.077	0.065	0.056	0.051
50–60	9	0.085	0.079	0.065	0.053	0.042	0.036	0.118	0.111	0.099	0.090	0.087	0.086
40–50	10	0.109	0.106	0.092	0.077	0.066	0.063	0.142	0.138	0.130	0.123	0.120	0.118
30–40	11	0.147	0.143	0.126	0.110	0.100	0.097	0.140	0.146	0.154	0.157	0.158	0.156
25–30	12	0.089	0.086	0.078	0.071	0.068	0.066	0.072	0.076	0.083	0.088	0.092	0.095
20–25	13	0.103	0.100	0.092	0.086	0.085	0.086	0.068	0.074	0.082	0.090	0.098	0.102
15–20	14	0.086	0.090	0.097	0.105	0.114	0.117	0.044	0.048	0.064	0.078	0.087	0.091
10–15	15	0.069	0.080	0.110	0.142	0.158	0.168	0.030	0.032	0.051	0.066	0.076	0.080
5–10	16	0.058	0.070	0.101	0.130	0.146	0.155	0.019	0.022	0.037	0.053	0.061	0.065
0–5	17	0.047	0.060	0.092	0.114	0.127	0.132	0.012	0.017	0.028	0.046	0.055	0.059

except at the higher values of K_{SL} where the assumptions used in the method become invalid. Even the theoretical values of I_3 and I_5 provide reasonably good estimates of δ_{SL}. The equations, used with Tables 1 and 2, will be accurate provided that V_E and γ_E are within the limits (i.e., above the curve for the vehicle's nominal β) shown in Fig. 2. Within these limits, a comparison of Finke's results and the method presented herein showed a maximum error of 75 ft out of a total dispersion of 2300 ft.

For a particular vehicle, accuracy may be improved by computing the I functions and weighting factors using 3DOF dispersion data for the specific vehicle and $V_E - \gamma_E$ region of

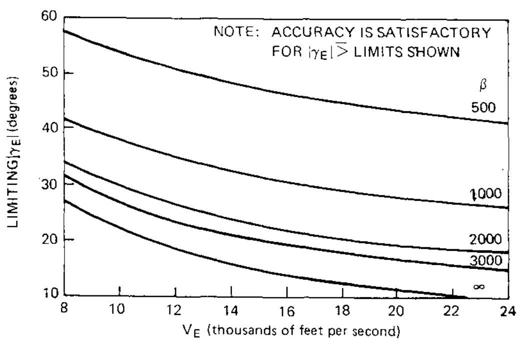

Fig. 2 Limiting values of γ_E.

interest. The empirical results may then be used for particular conditions at the firing point where computing facilities may be limited.

The method as presented is applicable for terminal altitudes (h_T) near 0 i.e., near sea level. By integrating Eqs. (18) and (34) given in the original text from conditions at re-entry to conditions at an arbitrary terminal altitude h_T, for a constant wind or a constant deviation in density it may be shown that the ratio of dispersion δ_T at h_T to the dispersion δ_{SL} at sea level varies in accordance with the equations

$$\delta_T/\delta_{SL} = I_3'/I_3 \quad \text{for wind}$$
$$\delta_T/\delta_{SL} = I_5'/I_5 \quad \text{for density} \qquad (3)$$

where I_3' and I_5' are the I_3 and I_5 functions given in Table 1 evaluated at $K_{SL} = -P_T/\beta \sin\gamma_E$ and P_T is the ambient pressure at h_T.

Equation (2) may be used to evaluate dispersion resulting from perturbations in axial force coefficient C_A or weight W by replacing $\Delta\rho/\rho$ by $\Delta C_A/C_A$ or by $-\Delta W/W$. In addition, the effect on dispersion of angle of attack (for a rolling vehicle) and perturbations in the speed of sound may be interpreted as changes in C_A and, therefore, may be evaluated by Eq. (2).

Reference

[1] R. G. Finke, "Re-entry Vehicle Dispersion Due to Atmospheric Variations," Research Paper P-506, Aug. 1969, Institute for Defense Analyses, Arlington, Va.